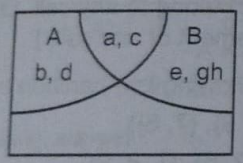
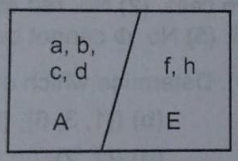
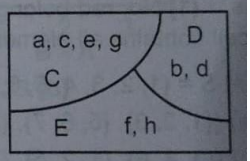
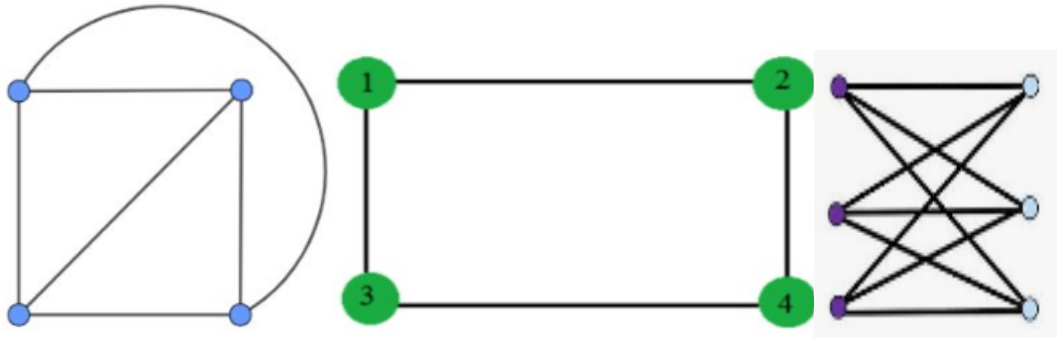
SIES Graduate School of Technology (Autonomous) NAAC A+ (Affiliated to University of Mumbai)	***End Semester Examination (R-24/R-25) SH 2026 Answer Key with marking scheme***
Branch: AIDS/AIML/CE	Course: Discrete Structures and Graph Theory
Year/ Semester: SE/III	Course code: AIDSC303/AIMLC303/CEC303
Time: 03 hours	Marks: 80

		Marks
Q. 1	All questions are compulsory.	
A.	$(p \rightarrow q) \wedge [\sim q \wedge (r \vee \sim q)]$ $= (\sim p \vee q) \wedge [\sim q \wedge (r \vee \sim q)]$ --- (Implication) $= (\sim p \vee q) \wedge [\sim q \wedge (\sim q \vee r)]$ --- (Commutativity) $= (\sim p \vee q) \wedge \sim q$ --- (Absorption) $= (\sim p \wedge \sim q) \vee (q \wedge \sim q)$ --- (Distributivity) $= (\sim p \wedge \sim q) \vee f$ --- (Complement) $= \sim p \wedge \sim q$ --- (Identity) $= \sim (p \vee q)$ --- (De Morgan's law)	05
B.	**Sol. : (i)** Consider $\{A, B\}$ (1) $A \cup B = \{a, b, c, d, e, g, h\}$ $\therefore f \in (A \cup B)$ $\therefore A \cup B \neq S$ (2) $A \cap B = \{a, c\} \neq \Phi$ $\therefore \{A, B\}$ is **not** a partition. [Fig. 2.31 (a)]  **Fig. 2.31 (a)**  **Fig. 2.31 (b)**  **Fig. 2.31 (c)** **(ii)** Consider $\{A, E\}$ (1) $A \cup E = \{a, b, c, d, f, h\}$; e and f do not belong to $A \cup B$. $\therefore A \cup B \neq S$ (2) $A \cap E = \Phi$ $\therefore \{A, E\}$ is **not** a partition. [Fig. 2.31 (b) above] **(iii)** Consider $\{C, D, E\}$ (1) $C \cup D \cup E = \{a, b, c, d, e, f, g, h\} = S$ (2) $C \cap D \cap E = \Phi$ $\therefore \{C, D, E\}$ **is** a partition. [Fig. 2.31 (c) above]	05

C.	Given recurrence relation is $a_n + 2a_{n-1} - 3a_{n-2} = 0$ Let $a_n = r^n$ $\therefore r^n + 2r^{n-1} - 3r^{n-2} = 0$ $\therefore r^{n-2}(r^2 + 2r - 3) = 0$ $\therefore r^2 + 2r - 3 = 0 \quad \therefore r = 1, -3$ $\therefore a_n = b_1 1^n + b_2 \times (-3)^n$ Putting $n = 0$, we get, $a_0 = b_1 + b_2 = 1$ Putting $n = 1$, we get, $a_1 = b_1 - 3b_2 = 2$ Solving, we get $b_1 = \frac{5}{4}$ and $b_2 = \frac{-1}{4}$ $\therefore$ Sol^n is $a_n = \frac{5}{4} \times 1^n - \frac{1}{4} \times (-3)^n$ $= \frac{5 - (-3)^n}{4}$	05
D.	**Sol. :** Since for every $a, b \in R$ there exists a unique element $a \circledast b = a + b + 1$ in R, hence, $\circledast$ is a binary operation in R. **G1 :** $(a \circledast b) \circledast c = (a + b + 1) \circledast c = (a + b + 1) + c + 1 = (a + b + c) + 2$ And $a \circledast (b \circledast c) = a \circledast (b + c + 1) = a + (b + c + 1) + 1 = (a + b + c) + 2$ $\therefore$ $(a \circledast b) \circledast c = a \circledast (b \circledast c)$ for all $a, b, c \in R$. $\therefore$ $\circledast$ is associative in R. **G2 :** To find identity element e, consider $a \circledast e = a$. But $a \circledast e = a + e + 1 \quad \therefore a = a + e + 1 \quad \therefore e + 1 = 0 \quad \therefore e = -1$. $\therefore$ -1 is an identity element. **G3 :** To find the inverse of a. Let b be the inverse of a. Then, $a \circledast b = e = -1 \quad \therefore a + b + 1 = -1 \quad \therefore b = -2 - a$ $\therefore$ $-2 - a$ is the inverse of a. Further, $a \circledast b = a + b + 1$ and $b \circledast a = b + a + 1$ $\therefore$ $a \circledast b = b \circledast a$. Hence, $\circledast$ is Abelian. Hence, G is an Abelian group.	05
E.	A graph in which every vertex has the same degree is called as Regular Graph. A graph G = (V, E) is complete bipartite, if i) G is bipartite, ii) G is simple, iii) V can be partitioned in two sets U and W, such that there is an edge between every vertex of U and every vertex of W.	05

	If a graph can be drawn in such a way that no two edges of the graph intersect with each other, then such a graph is called Planar Graph.  	
F.	Let $P(n): 1+2+2^2+\cdots+2^n = 2^{n+1}-1$ Step-I :- Let $n=1$: Then LHS $= 3$ and RHS $= 2^{1+1}-1 = 2^2-1 = 4-1 = 3 =$ LHS $\therefore P(1)$ is true. Step-II :- Assume that $P(k)$ is true. $\therefore 1+2+2^2+\cdots+2^k = 2^{k+1}-1$ Now to prove that $1+2+2^2+\cdots+2^{k+1} = 2^{k+2}-1$ Now LHS $= 1+2+2^2+\cdots+2^{k+1} = 1+2+2^2+\cdots+2^k+2^{k+1}$ $= 2^{k+1}-1+2^{k+1} = 2\times 2^{k+1}-1 = 2^{k+2}-1 =$ RHS $\therefore$ Result is true for $k=n+1$	05
Q.2A i)	Sol. : (i) Since $x-x=0$, $x-x$ is divisible by 4. $\therefore$ R is reflexive. (ii) If $x-y$ is divisible by 4 then $y-x$ is also divisible by 4. $\therefore$ R is symmetric. (iii) If $x-y$ is divisible by 4, then $x-y=4n$, say if $y-z$ is divisible by then $y-z=4m$, say. Now, $x-z=(x-y)+(y-z)=4n+4m=4(m+n)$ $\therefore$ $x-z$ is divisible by 4. $\therefore$ R is transitive. $\therefore$ R is an equivalence relation. To find the equivalence classes, we take 0. Then, $E(0)=\{x \mid (x-0)$ is divisible by 4 $\}$. In words $E(0)$ means the set of all integers which are such that $(x-0)$ *i.e.*, x is divisible by $\therefore$ $E(0)=\{\ldots, -8, -4, 0, 4, 8, \ldots\}$ $E(1)=\{x \mid (x-1)$ is divisible by 4 $\}$ In words, $E(1)$ means the set of integers which are such that $x-1$ is divisible by 4, *i.e.*, $(x-1)$ is a multiple of 4. *i.e.* x is a multiple of 4 plus 1. $\therefore$ $E(1)=\{\ldots, -11, -7, -3, 1, 5, 9, \ldots\}$ Similarly, by adding 2, 3, 4 to the elements of $E(0)$, we get $E(2)=\{\ldots, -6, -2, 2, 6, 10, \ldots\}$ $E(3)=\{\ldots, -5, -1, 3, 7, 11, \ldots\}$	05

ii)	$\therefore$ (i) We have $g \circ h = g \circ (3x) = 3x - 2$ $\therefore$ $f \circ g \circ h = f \circ (3x - 2) = 3x - 2 + 2 = 3x$ (ii) We have, $g \circ f = g(x + 2) = x + 2 - 2 = x$ $\therefore$ $h \circ g \circ f = h(x) = 3x$ (iii) We have, $f \circ f = f \circ (x + 2) = x + 2 + 2 = x + 4$	05
	OR	
Q.2B.	Find transitive closure ... (M.U. 1996) **Sol. : Step 1 :** We write the matrix $\mathbf{M}_R$ of the given relation R and denote it by W_0. $W_0 = \mathbf{M}_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ **Step 2 :** We write a blank matrix of order 4, denote it by W_1 and transfer all 1's from W_0 to W_1. $W_1 = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & 1 \\ 1 & 1 & \dots & \dots \\ \dots & 1 & 1 & 1 \end{bmatrix}$ Column one $\rightarrow p_i = 1, 3$; Row one $\rightarrow q_j = 1, 2, 4$. So, we put 1 in (p_i, q_j) (1, 1), (1, 2), (1, 4), (3, 1), (3, 2), (3, 4). $\therefore$ $W_1 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ **Step 3 :** $W_2 = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \dots & 1 & 1 & 1 \end{bmatrix}$ Column two $\rightarrow p_i = 1, 3, 4$. Row two $\rightarrow q_j = 4$. So we put 1 in (p_i, q_j), (1, 4), (3, 4), (4, 4) positions if there is no one already. $\therefore$ $W_2 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ **Step 4 :** $W_3 = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \dots & 1 & 1 & 1 \end{bmatrix}$	10

Column three → $p_i = 4$. Row three → $q_j = 1, 2, 4$.

So, we put 1 in (p_i, q_j), (4, 1), (4, 2), (4, 4) positions, if there is no one already.

$$\therefore \quad W_3 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Step 5 :

$$W_4 = \begin{bmatrix} 1 & 1 & \dots & 1 \\ \dots & \dots & \dots & 1 \\ 1 & 1 & \dots & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Column four → $p_i = 1, 2, 3, 4$. Row four → $q_j = 1, 2, 3, 4$.

So, we put 1 in all positions (p_i, q_j), (1, 1), (1, 2), ..., (4, 3), (4, 4) positions.

$$\therefore \quad W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\therefore \quad R^{\infty} = \{ (1, 1) (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4) \}$$

Q.3A

(a) {1, 3, 9, 18}, (b) {3, 5, 30}, ((M.U. 2000)

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Sol. : (a) The partial order relation "divides" on the set {1, 3, 9, 18} is

$R = \{(1, 1), (1, 3), (1, 9), (1, 18), (3, 3), (3, 9), (3, 18), (9, 9), (9, 18), (18, 18)\}$

Matrix of the relation is

$$M_R = \begin{array}{c} \\ 1 \\ 3 \\ 9 \\ 18 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 3 & 9 & 18 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

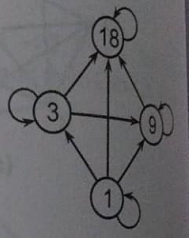


Fig. 5.10 (a)

We first find the digraph [Fig. 5.10 (a)].

Step 1 : Delete all the loops [Fig. 5.10 (b)].

Step 2 : Delete the edges that are implied by transitivity. Delete the edges from 1 to 9, 1 to 18 and 3 to 18 [Fig. 5.10 (c)].

Step 3 : Drop the arrow heads [Fig. 5.10 (d)]. The result is the Hasse diagram [Fig. 5.10 (e)]. This is a chain. See the definition (d), page 5-4.

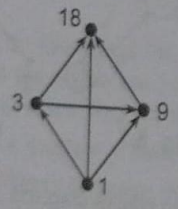
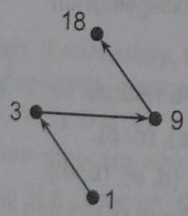
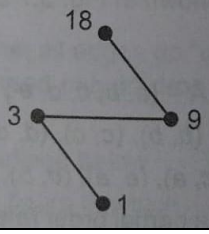
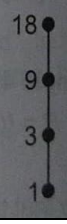


Fig. 5.10 (b)







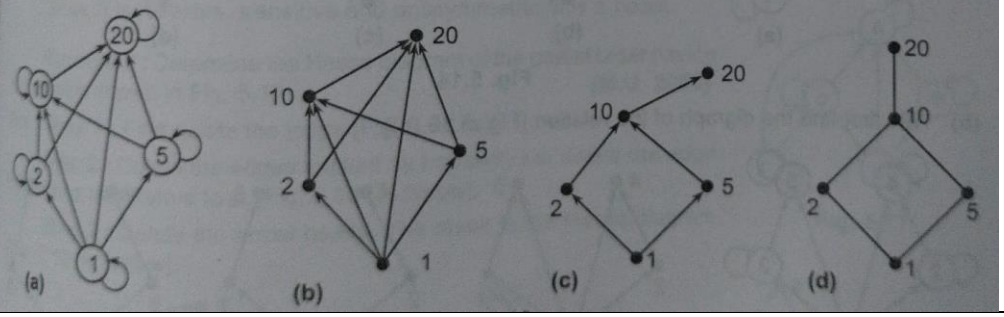
(c) The partial order relation "divides" on the set {1, 2, 5, 10, 20} is

$R = \{(1, 1), (1, 2), (1, 5), (1, 10), (1, 20), (2, 2), (2, 10)$
$(2, 20), (5, 5), (5, 10), (5, 20), (10, 10), (10, 20), (20, 20)\}$

The matrix of the relation is

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 5 \\ 10 \\ 12 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 5 & 10 & 20 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

We first find the digraph [Fig. 5.12 (a)].



OR

Q.3B

Example 2 : Show that

Sol. : The set of all divisions is the set $A = \{1, 2, 5, 7, 10, 14, 35, 70\}$

The Hasse diagram of the poset is shown in Fig. 5.80.

Fig

As in the above example, we get the following tables.

$\vee$	1	2	5	7	10	14	35	70
1	1	2	5	7	10	14	35	70
2	2	2	10	14	10	14	70	70
5	5	10	5	35	10	70	35	70
7	7	14	35	7	70	14	35	70
10	10	10	10	70	10	70	70	70
14	14	14	70	14	70	14	70	70
35	35	70	35	35	70	70	35	70
70	70	70	70	70	70	70	70	70

$\wedge$	1	2	5	7	10	14	35	70
1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2
5	1	1	5	1	5	1	5	5
7	1	1	1	7	1	7	7	7
10	1	2	5	1	10	2	5	10
14	1	2	1	7	2	14	7	14
35	1	1	5	7	5	7	35	35
70	1	2	5	7	10	14	35	70

Since every pair of elements has a LUB and GLB the set is a lattice.

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Q.4A	$n(R)$ = the number of cars having radio = 12 $n(W)$ = the number of cars having power windows = 11 $n(A \cap W) = 5$, $n(A \cap R) = 9$, $n(R \cap W) = 4$, $n(A \cap R \cap W) = 3$, $n(\overline{A \cup R \cup W}) = 2$. (i) The number of cars having power windows only $n(W - A - R) = n(W \cap \overline{A} \cap \overline{R}) = n(W) - n(W \cap A) - n(W \cap R) + n(W \cap A \cap R)$ $= 11 - 5 - 4 + 3 = 5$ (ii) The number of cars having air conditioning only $n(A - R - W) = n(A \cap \overline{R} \cap \overline{W}) = n(A) - n(A \cap R) - n(A \cap W) + n(A \cap R \cap W)$ $= 15 - 9 - 5 + 3 = 4$ (iii) The number of cars having radio only $n(R - A - W) = n(R \cap \overline{A} \cap \overline{W}) = n(R) - n(R \cap A) - n(R \cap W) + n(R \cap A \cap W)$ $= 12 - 9 - 4 + 3 = 2$ (iv) The number of cars having radio and power windows but not air-conditioning $n(R \cap W \cap \overline{A}) = n(R \cap W) - n(R \cap W \cap A)$ $= 4 - 3 = 1$ (v) The number of cars having air-conditioning and radio but not power windows $n(A \cap R \cap \overline{W}) = n(A \cap R) - n(A \cap R \cap W)$ $= 9 - 3 = 6$ (vi) The number of cars having only one option $= n(A - R - W) + n(R - A - W) + n(W - A - R)$ $= 4 + 2 + 5 = 11$	10
OR		
Q.4B i)	i) 0·5, ii) $\frac{5}{6}$, iii) 0·8	05
ii)	P(selecting true coin) = $p_1 = \frac{3}{4}$, P(selecting false coin) = $p_2 = \frac{1}{4}$ p_1' = P(getting all four heads with true coin) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$ p_2' = P(getting all four heads with false coin) $= 1 \times 1 \times 1 \times 1 = 1$ ∴ Required probability = $\frac{p_2 p_2'}{p_1 p_1' + p_2 p_2'} = \frac{\frac{1}{4} \times 1}{\frac{3}{4} \times \frac{1}{16} + \frac{1}{4} \times 1} = \frac{16}{19}$	05

Q.5A	8 and multiplication modulo ... set A with addition modulo ... (M.U. 2022) **Sol. :** The addition and multiplication tables for A modulo 8 are given below.	10

The Addition Table 1

⊕	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5	6	7	0	1	2
4	4	5	6	7	0	1	2	3
5	5	6	7	0	1	2	3	4
6	6	7	0	1	2	3	4	5
7	7	0	1	2	3	4	5	6

The Multiplication Table 2

⊗	0	1	2	3	4	5	6	7
0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7
2	0	2	4	6	0	2	4	6
3	0	3	6	1	4	7	2	5
4	0	4	0	4	0	4	0	4
5	0	5	2	7	4	1	6	3
6	0	6	4	2	0	6	4	2
7	0	7	6	5	4	3	2	1

(G) R_1 : To prove that $(R_8 +_8)$ is a commutative group : $+_8$ is a closed operation since table 1 belongs to the set A.

(G_1) $+_8$ is an associative operation.

For example, $2_{+8} + (3_{+8} + 4_{+8}) = 2_{+8} + (7_{+8}) = 1$

and $(2_{+8} + 3_{+8}) + 4_{+5} = 5_{+8} + 4_{+8} = 1$

(G_2) 0 is identity.

From column 1 of Table 1, we see that $1 + 0 = 1$, $2 + 0 = 2$, $3 + 0 = 3$, ..., etc.

From row 1 of Table 1, we see that $0 + 1 = 1$, $0 + 2 = 2$, $0 + 3 = 3$, ..., etc.

(G_3) Every element has an inverse.

From the table, we see that, $1 +_8 7 = 0$, $2 +_8 6 = 0$, $3 +_8 5 = 0$, $4 +_8 4 = 0$, ..., etc.

G is a commutative group.

From Table 1, we see that, $2 +_8 3 = 6$ and $3 +_8 2 = 6$,

$4 +_8 5 = 4$ and $5 +_8 4 = 4$, ..., etc.

R_2 : To prove that ⊗ is associative

i.e., $(a \otimes_8 b) \otimes_8 c = a \otimes_8 (b \otimes_8 c)$ for all $a, b, c \in R$.

From the Table 2, we see that $(2 \otimes_8 3) \otimes_8 4 = 6 \otimes_8 4 = 0$ and $2 \otimes_8 (3 \otimes_8 4) = 2 \otimes_8 4 = 0$

Also, $(3 \otimes_8 4) \otimes_8 5 = 4 \otimes 5 = 4$ and $3 \otimes_8 (4 \otimes_8 5) = 3 \otimes 4 = 4$

OR

Q.5B	**Sol. :**	10

⊕	0000000	0010110	0101000	0111110	1000101	1010011	1101101	1111011
0000000	0000000	0010110	0101000	0111110	1000101	1010011	1101101	1111011
0010110	0010110	0000000	0111110	0101000	1010011	1000101	1111011	1101101
0101000	0101000	0111110	0000000	0010110	1101101	1111011	1000101	1010011
0111110	0111110	0101000	0010110	0000000	1111011	1101101	1010011	1000101
1000101	1000101	1010011	1101101	1111011	0000000	0010110	0101000	0111110
1010011	1010011	1000111	1111011	1101101	0010110	0000000	0111110	0101000
1101101	1101101	1111011	1000101	1010011	0101000	0111110	0000000	0010110
1111011	1111011	1101101	1010011	1000101	0111110	0101000	0010110	0000000

(i) From the diagonal elements of the above table we see that (0000000) is an identity and it belongs to N.

(ii) From the table we also see that if x, y belong to N then $x \oplus y$ belongs to N.

(iii) Every element is its inverse.

$\therefore$ N is a subgroup of B^7 and the given encoding function is a group code. The minimum distance is 2. Hence, the code can detect 1 or less errors.

Q.6A i)	**Sol. :** We first see that both the graphs have the same number of vertices (6) and the same number of edges (9). In G_1 all the vertices are of degree 3 and in G_2 also all the vertices are of degree 3. Consider the correspondence $v_1 \rightarrow u_1$, $v_2 \rightarrow u_2$, $v_3 \rightarrow u_3$, $v_4 \rightarrow u_4$, $v_5 \rightarrow u_5$, $v_6 \rightarrow u_6$. Hence G1 and G2 are isomorphic.	05
ii)	**Sol. :** Both the graphs have 5 vertices. Both the graphs have 6 edges. **(M.U. 2005)** But in graph G_2 there is one vertex a' with degree 4 and one vertex e′ with degree 1. There is no vertex with degree 4 and no vertex with degree 1 in G_1. ∴ The graphs are not isomorphic.	05
	OR	
Q.6B i)	Eulerian path and Eulerian circuit. **Sol. :** Since all vertices are of even degree the graph contains Euler's circuit. Eulerian path : *A, B, C, D, E, F.* Eulerian Circuit : *A B D A E D C B E F A*	05
ii)	The graph G_2 has 6 vertices *a, b, c, d, e, f* with degree 3, 4, 2, 4, 3, 4. The sum of the degrees of any two vertices is greater than or equal to $(n - 1) = 5$. Hence, by theorem 1 there is a Hamiltonian circuit and hence it is a Hamiltonian graph. π : *a, f, b, c, d, e, a.* Hamiltonian path is π : *a, f, b, c, d, e.*	05

*********** All the Best***********